S.-T. Yau College Student Mathematics Contests 2022

Oral Exams in Geometry and Topology

Individual (3 problems)

1. Let M be a closed orientable manifold with dimension n, and let S^n be the n dimensional sphere.

(a) What is the degree $\deg(f)$ of a continuous map $f : M \to S^n$;

(b) For an non-negative integer $k \in \mathbb{N}$ construct a map $f : M \to S^n$ with $\deg(f) = k$.

2. Construct a Riemannian metric on $\mathbb{C}P^2 \# \mathbb{C}P^2$ with nonnegative sectional curvature.

3. Let M^n be a closed and embedded minimal hypersurface in the unit sphere S^{n+1}. Then the first eigenvalue $\lambda_1(M^n)$ of M^n satisfies

$$\lambda_1(M^n) \geqslant \frac{n}{2}.$$

S.-T. Yau College Student Mathematics Contests 2022

Oral Exams in Geometry and Topology

All-round (2 problems)

1. Prove that the tangent bundle of the product $S^2 \times S^3$ is trivial.

2. Any closed (compact and without boundary), oriented and connected even-dimensional Riemannian manifold with positive sectional curvature is simply connected.